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THE POTOMAC ESTUARY MATHEMATICAL MODEL

Leo J. Hetling, et al

Federal Water Pollution Control Administration
Annapolis, Maryland

March 1969

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Chesapeake Technical Support Laboratory
Middle Atlantic Region
Federal Water Pollution Control Administration
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Technical Report No. 7

THE POTOMAC ESTUARY

MATHEMATICAL MODEL

by

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March 1969

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INTRODUCTION

A systems analysis approach has been undertaken by the Federal Water Pollution Control Administration (FWPCA) in investigating the water quality responses in the Potomac River Basin [1]. The analyses included the effects of low flow augmentation, wastewater diversions, water supply withdrawals, and increased degrees of wastewater treatment on water quality in the upper estuary.

Several techniques or mathematical models* capable of simulating the response of water quality in an estuary were available when the study of the Potomac Estuary was undertaken in 1965. After an investigation of the modeling systems available, the segmented estuary model developed by Dr. Robert Thomann [2] was selected as the one which most closely conformed to the requirements of the study.

The segmented model is highly flexible and capable of being utilized to describe almost any pollutant. Its accuracy is adequate for engineering design purposes. Properly programmed, the digital computer makes possible solutions of the systems available in minutes at a relatively small cost.

* A key element in any systems study is a model capable of describing or simulating the system of interest.

THE MODEL

In order to give the non-engineers among you some idea of what a mathematical model is, I would like to quote from a description of models I previously prepared [3]:

"Anyone who has a checking account works with a mathematical model. All deposits are positives and all withdrawals are negatives. Monthly service charges and a charge per check further completes the model. Using technical jargon, we can say that the model has a lower limit (or bound - using the correct mathematical term) in that a negative balance cannot, in theory, exist. With a knowledge of all the inputs (deposits and withdrawals), the initial conditions (original balance) and other sources and sinks (service charges), it is possible to compute the balance at any time (a transient model).

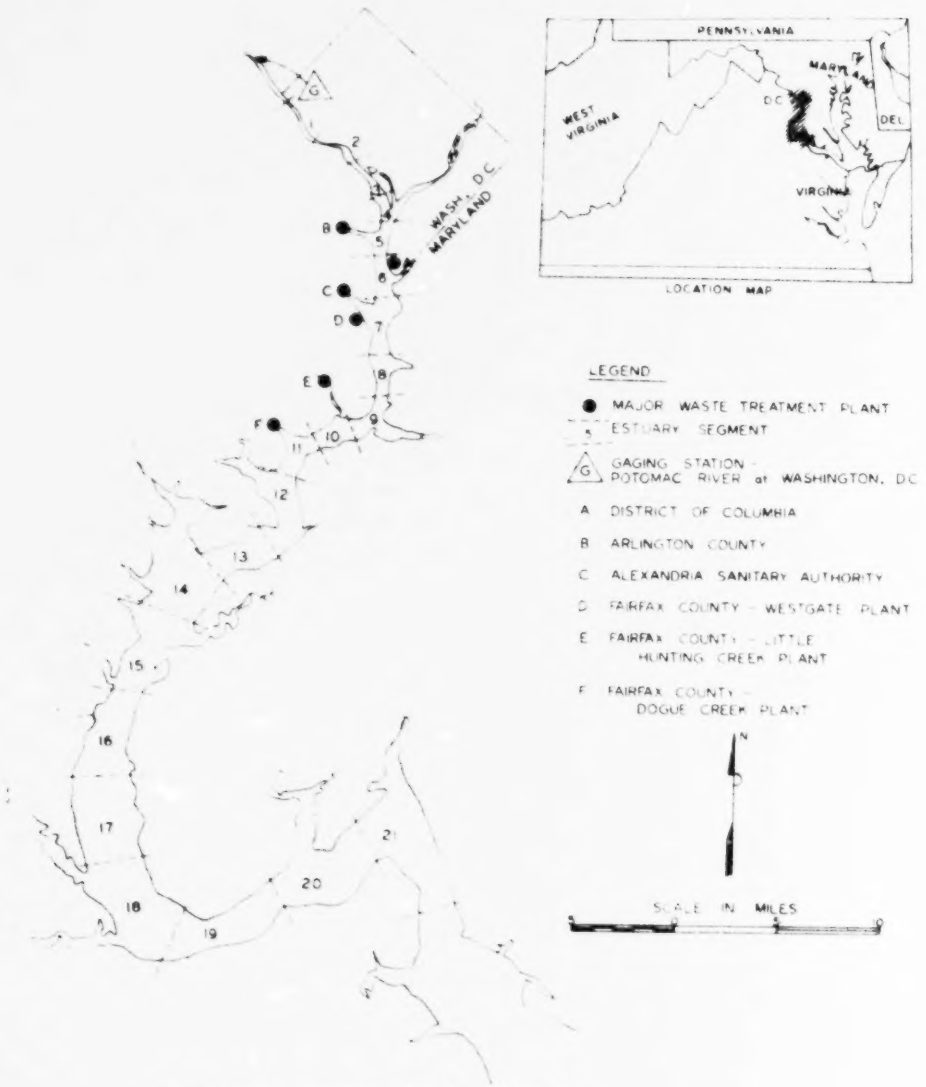
"In the same way, by proper bookkeeping and a knowledge of bounds or limits, inputs, initial condition, sources and sinks, it is possible to develop a set of equations which describe water quality in the estuary. The number of computations, of course, in this system when compared with a checking account is enormous. To solve such a system would require many man-years. But, luckily, most of the computations are routine addition, subtraction, and multiplication. Within recent years, digital

computers have been developed which can do such routine computations in a fraction of a second. Properly instructed (programmed), the digital computer makes possible solutions of the system available in minutes.

"A key step in the development of any model is its verification; i.e., to check and see if the model truly simulates water quality. This involves a comparison of measured or observed values with computed values.

"A checking account is verified every so often when a statement is received from the bank. When we compare the balance in our checkbook with the balance on the bank statement, we find time after time that we have made a mistake. More often than not, it is a blunder; an input (deposit or withdrawal) was forgotten, the bank service charge was changed, or perhaps even the bank made a mistake."

In a more technical sense, the mathematical model for the Potomac Estuary was constructed by dividing the estuary into a finite number of discrete segments as shown in Figure 1. The segments were made small (approximately two miles in length) near the critical portion of the estuary where the rate of changes of water quality is greater and longer in the lower sections where there are small changes



POTOMAC RIVER STUDY AREA

Figure 1

in water quality. The assumption is made that the water quality in each segment is homogeneous if the segments are properly selected, and if they are made small enough, this is a proper assumption.

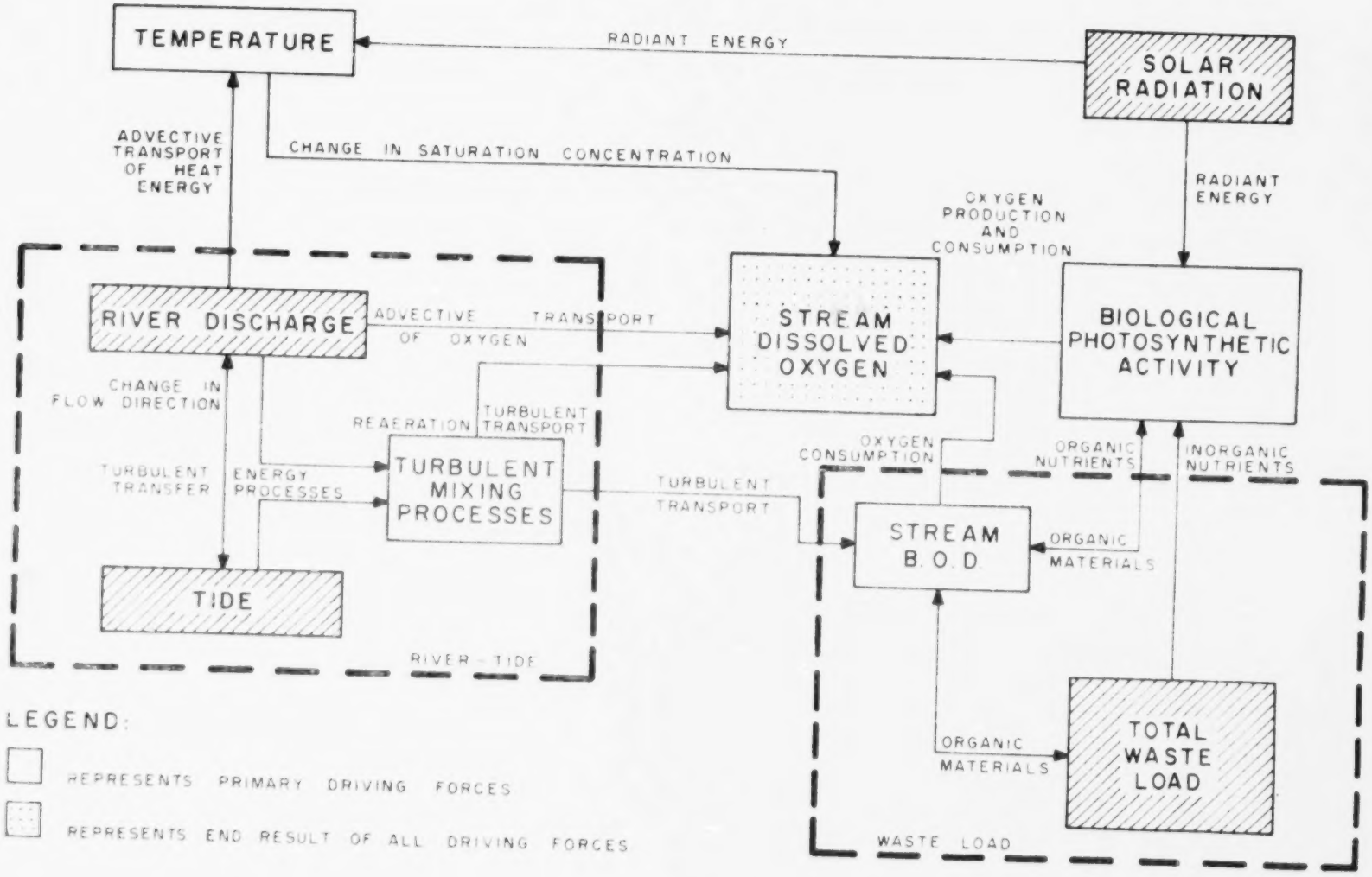
For each water quality parameter we started by writing the equations utilizing all the factors which we felt would affect the given water quality parameter. The upper Potomac Estuary model for a non-conservative pollutant such as a biochemical oxygen demand (BOD) consists of a system of 21 equations, each describing a mass BOD balance for the 21 segments of the estuary.

The mass balance over each of the 21 segments shown in Figure 1 includes terms describing changes in BOD concentration caused by advection, dispersion, decay, and BOD added by the various sources. The system of 21 linear first order, non-homogeneous, ordinary differential equations which describes the changes is then solved simultaneously by numerical methods using a digital computer.

Dissolved oxygen (DO) is probably one of the most important parameters to know when talking about the water quality of the estuary. It is also the most difficult to model, since DO can enter or leave the estuary in so many different ways. Figure 2 will give you an idea of the system which

affects DO concentration in an estuary. Factors such as benthic uptake of oxygen, nitrification, and photosynthesis further complicate the DO balance in the Potomac Estuary.

For simulating the DO in the upper estuary, the model consists of 42 equations, 21 describing a mass balance for the ultimate oxygen demand (UOD) and 21 equations for the dissolved oxygen. Vertical and lateral homegeneity in each segment are assumed.



FACTORS AFFECTING STREAM DISSOLVED OXYGEN CONCENTRATION

MODEL VERIFICATION USING DYE STUDIES

I would like to describe a large-scale study which was carried out to verify the model in order to give you a better feel for its capabilities [4].

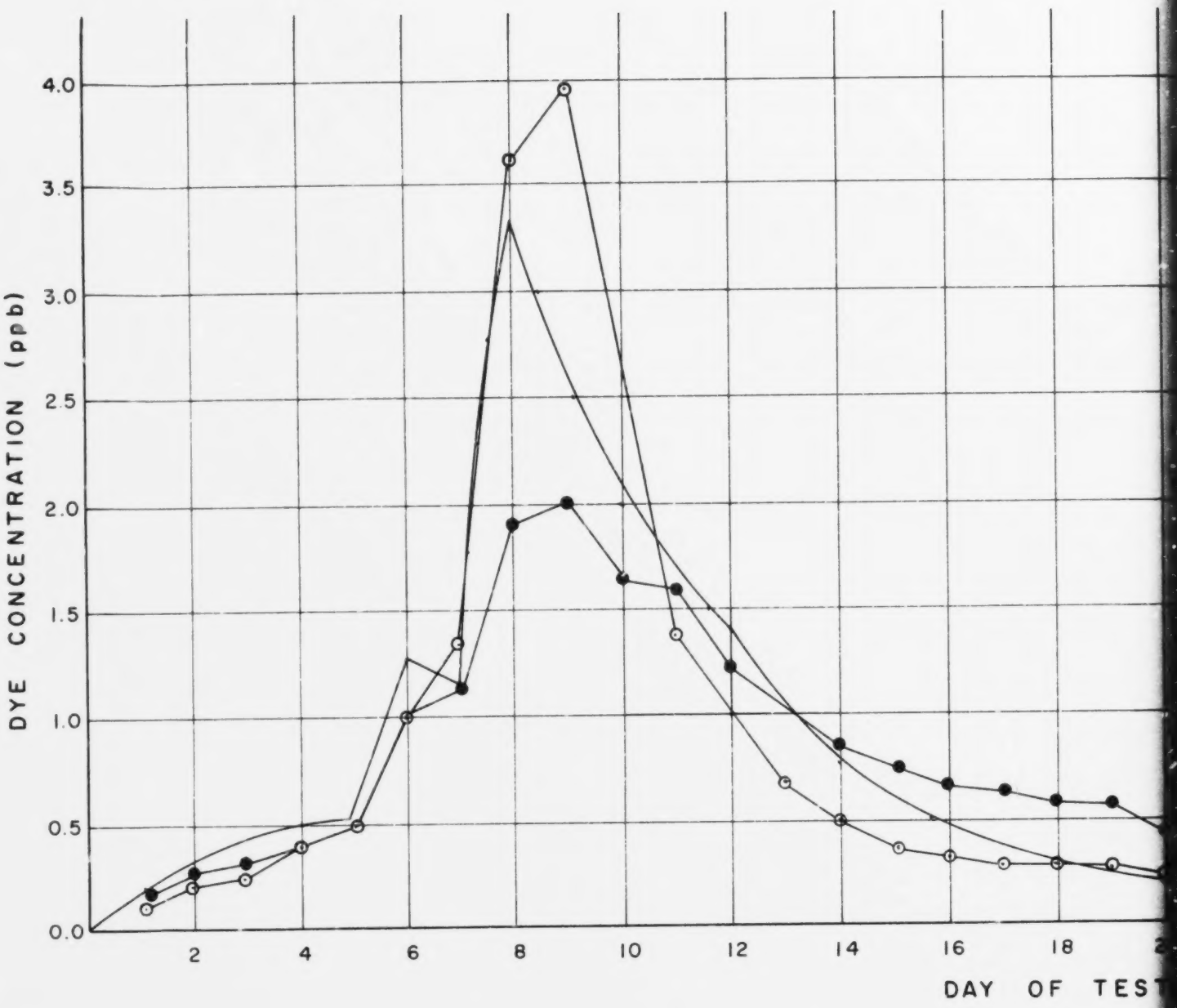
Most natural water quality parameters are involved in so many complex chemical, biochemical, and physical reactions that they do not lend themselves to a good simple test of the model. Therefore, our first attempt at verification was a dye diffusion study. We injected a semi-conservative dye (Rhodamine WT) into the estuary through the effluent pipes of the District of Columbia's Water Pollution Control Plant at a known rate. See Figure 1 for the location of the waste-water treatment plant.

Figure 3 shows the resulting average dye concentration in the estuary measured opposite the plant outfall. The solid center line shows the concentration of dye predicted by the model, while the other two lines represent the actual measured concentrations at high and low tide.

The matching of the observed data and calculated data did not, of course, occur on our first try with the initial model coefficients. Many repetitive runs of the model were required to get the match you see here. For each repetitive run, a change in a coefficient was made in order to bring the predictive concentrations closer to the measured values.

The close agreement between the measured and calculated dye concentrations found in the above study shows that the segmented estuary model can satisfactorily describe water quality which will result from the introduction of a simple soluble pollutant into the estuary if the correct coefficients are known.

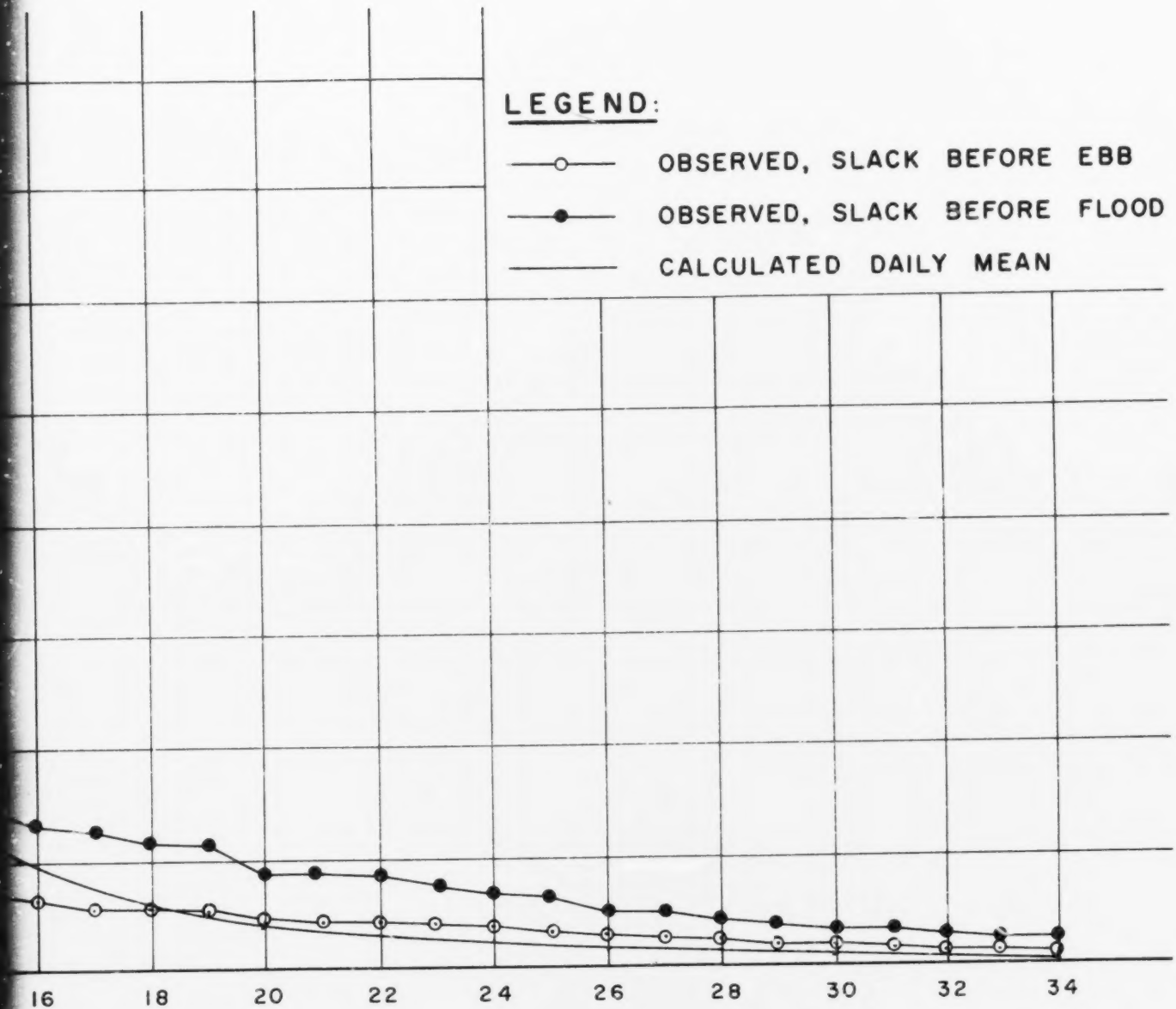
In contrast to the thinking of many people, mathematical models do not relieve the engineer from making assumptions and decisions; what it does do is force him to test the validity of these assumptions by comparing their compatibility with actual measured results.



POTOMAC EST

OBSERVED AND CALCULATED DYE CO

A



MAC ESTUARY
LATED DYE CONCENTRATIONS vs TIME

B

Figure 3

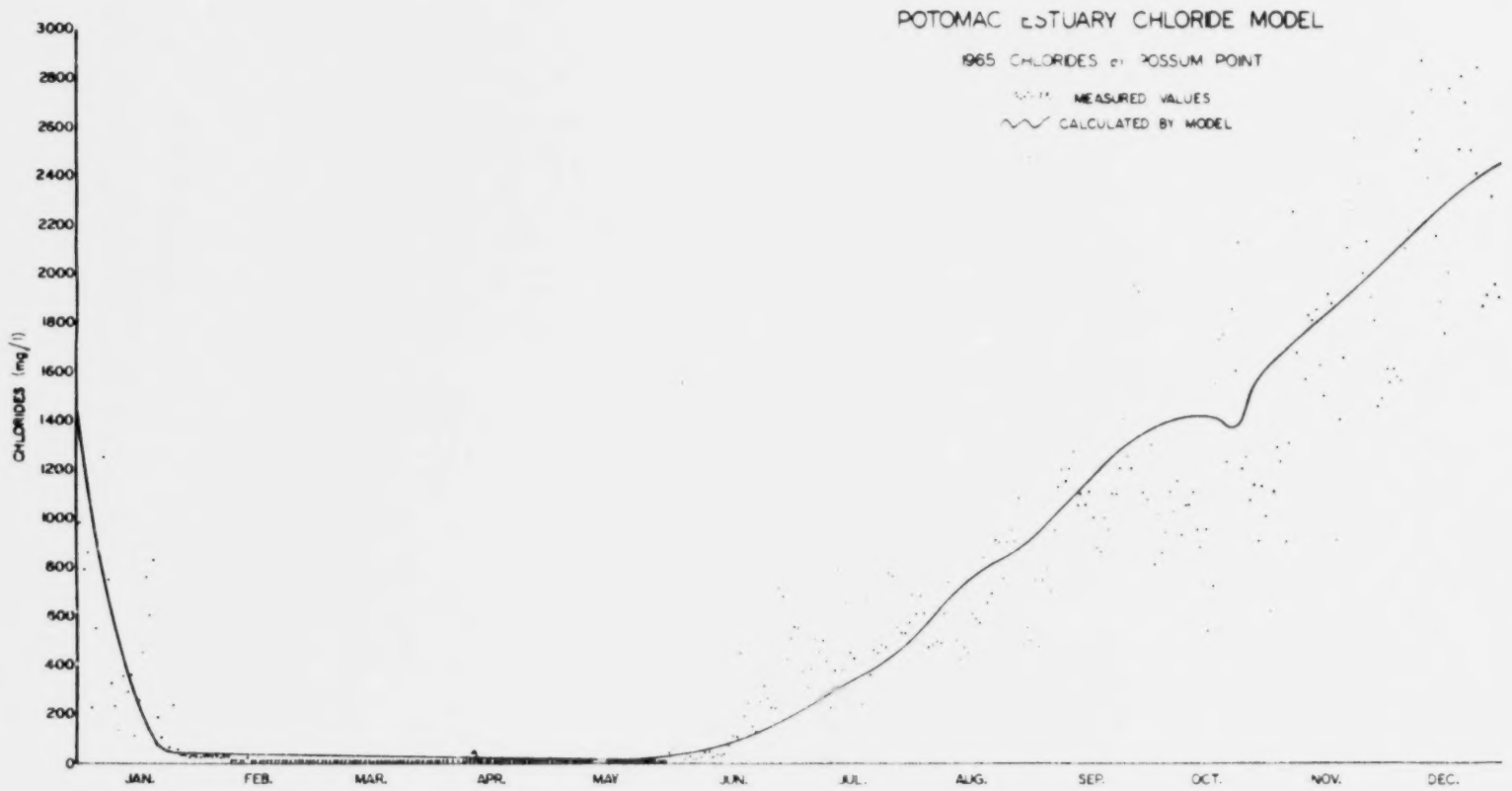
WATER QUALITY MODELING RESULTS

Now for some germane water quality parameters, I would like to show our results in predicting chlorides and dissolved oxygen.

Chlorides

Chlorides enter the estuary mainly by diffusion upstream from the Chesapeake Bay. The rate of diffusion and extent of intrusion depends primarily on the tidal currents and river flow. Figure 4 shows the observed and calculated variation in chlorides in the estuary at Possum Point near Quantico, Virginia. In the figure you can see that the concentration of chlorides decreased dramatically during the late winter and spring months when the river flow was high and increased in the late fall and early winter when the river flow was low.

The chloride concentrations were also used as a natural tracer in establishing the dispersion coefficients for the portions of the estuary not within the 1965 dye release as previously described [5]. In Figure 5, the dispersion coefficients are presented as determined by the dye studies and from a study of the chloride intrusion. Thus, the model can be used to predict both a given water quality parameter and to establish model coefficients which can be utilized for modeling other water quality parameters such as DO.


POTOMAC ESTUARY CHLORIDE MODEL
1965 CHLORIDES at POSSUM POINT
MEASURED VALUES
CALCULATED BY MODEL
CHLORIDES (mg/l)
3000
2800
2600
2400
2200
2000
1800
1600
1400
1200
1000
800
600
400
200
0
JAN.
FEB.
MAR.
APR.
MAY
JUN.
JUL.
AUG.
SEP.
OCT.
NOV.
DEC.

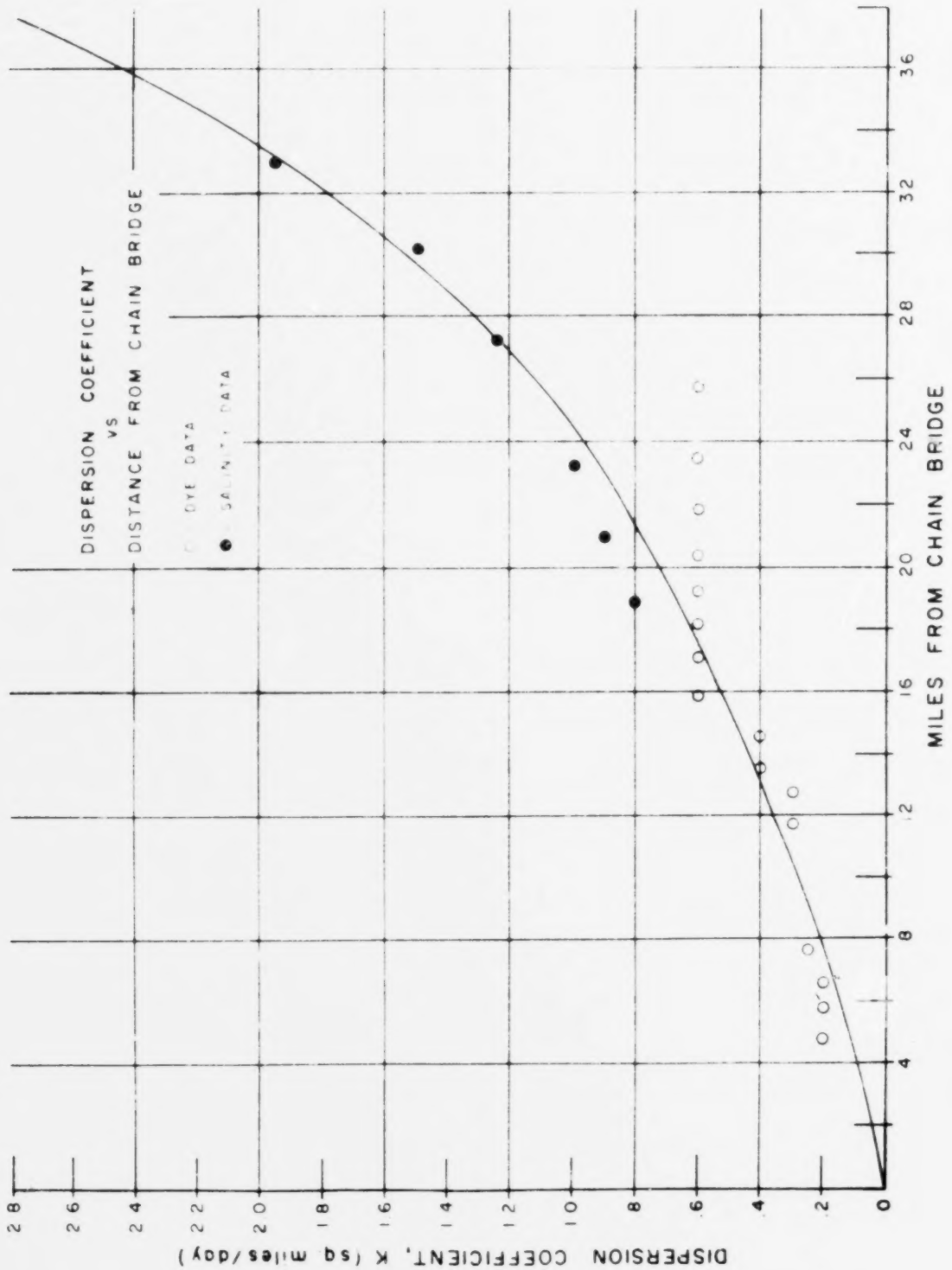

DISPERSION COEFFICIENT
VS
DISTANCE FROM CHAIN BRIDGE
DYE DATA
SALINITY DATA
DISPERSION COEFFICIENT, K (sq miles/day)
MILES FROM CHAIN BRIDGE
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
4
8
12
16
20
24
28
32
36

Figure 5

Dissolved Oxygen

Our results in modeling DO have been very encouraging even though there are many factors affecting the dissolved oxygen in an estuary as indicated previously [3]. The capability of the model to predict the effects of the addition of wastewater loadings on DO in the Potomac Estuary can readily be seen when Figures 6 and 7 are compared.

Presented in Figure 6 is the observed and predicted DO concentration for the Potomac Estuary in 1965 near Memorial Bridge which is upstream from the major wastewater discharges. The observed and predicted average DO for the summer months is about 6 mg/l. Figure 7 shows the observed and predicted DO in the estuary in 1965 at Woodrow Wilson Memorial Bridge which is within the tidal excursion range of the major discharges. The observed and predicted average DO for the summer months is about 2.0 mg/l.

It should be noted that only average concentrations are obtained from the output of the model. The "to and fro" oscillations caused by tidal movement and diurnal fluctuations as a result of phytoplankton activity can create significant variations from the mean predicted concentrations. Nevertheless, the ability to predict the average daily concentration of DO (normally used in design) has been adequately demonstrated.

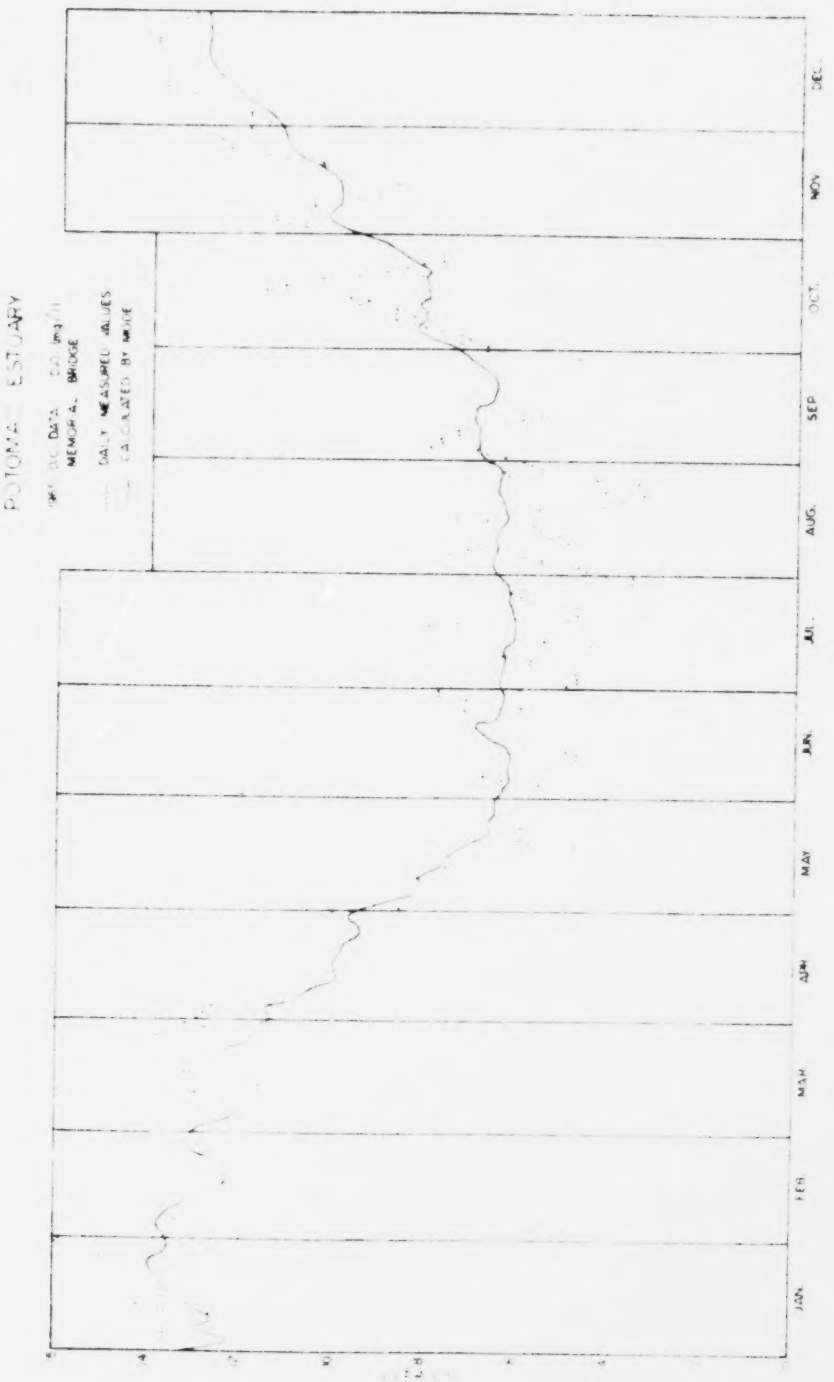

POTOMAC ESTUARY
DEC.
NOV.
OCT.
SEP.
AUG.
JUL.
JUN.
MAY
APR.
MAR.
FEB.
JAN.

Figure 6

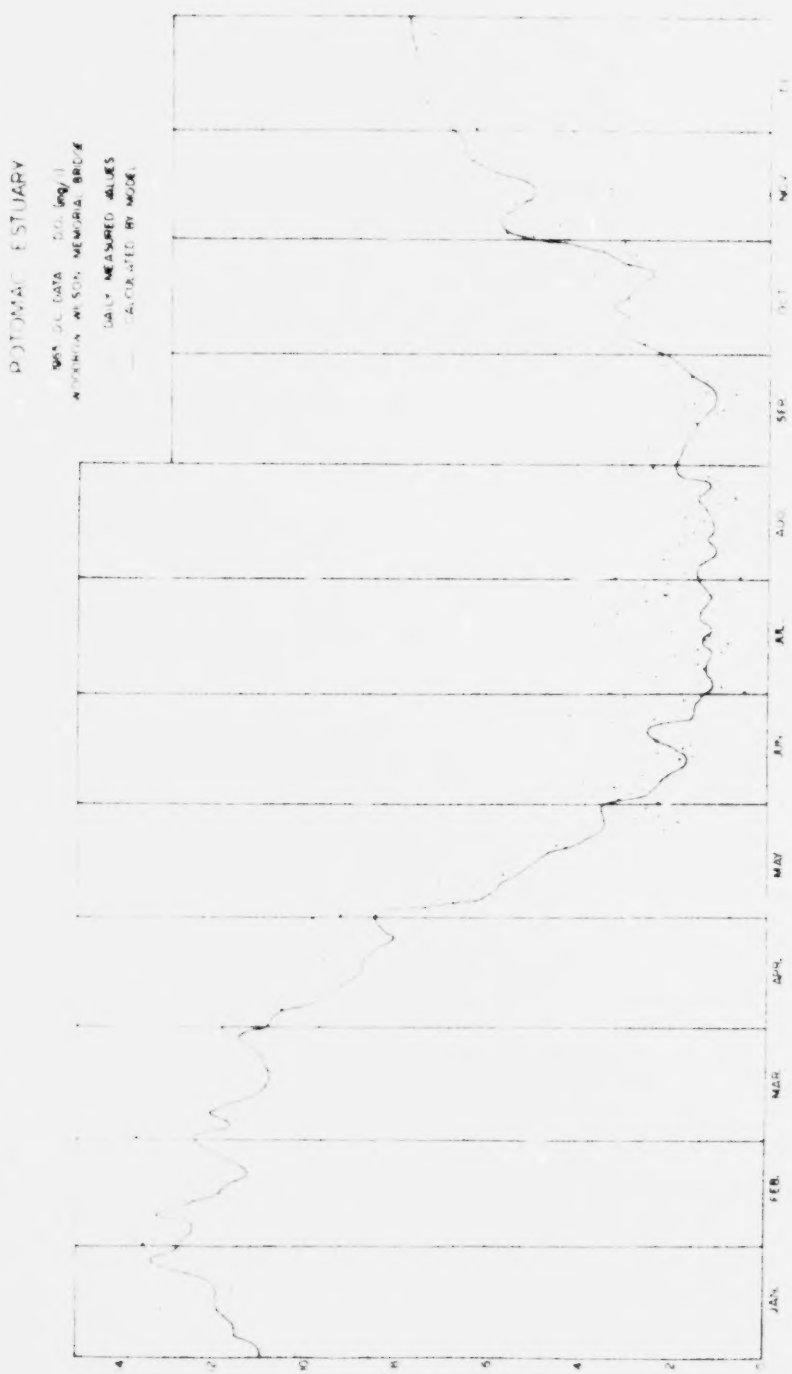


Figure 7

USES OF THE POTOMAC ESTUARY MODEL

I think that following this somewhat technical description of the model, it might be helpful to discuss the uses made of the model for the Potomac Estuary and thus get a feel for how models and people interact.

Up to the present time, the model described has been utilized in a variety of ways in the Potomac Estuary water quality management process as summarized below:

1. In its studies to determine a range of feasible alternative plans to improve water quality in the estuary, the Water Quality Sub-Task Force of President Johnson's "Project Potomac" [ ] made extensive use of the model.

2. Results from the model were used by Dr. Robert Davis, of Resources For the Future, in his study of the effects of advances in technological software on the planning process. Dr. Davis recently published a book entitled "The Range of Choice in Water Resources Management: A Study of the Potomac Estuary" which resulted from this application.

3. The model has been used to determine the feasibility of using the upper estuary as a future emergency water supply for the Washington Metropolitan Area [ ]. On the basis of salinity considerations only, this has been shown to be a feasible alternative. The economics and aesthetics of such

a program have not yet been investigated.

4. It has been used at the request of various states to assist in developing their long-range water quality control plans.

5. It is continuously utilized by FWPCA as a tool for planning for and analyzing the results of field studies. Thus, it has proven extremely valuable in development work in such areas as nitrification, nutrient transport and algal control. The results of these studies are, in turn, put back into the model in the form of better estimations of the model coefficients and thus improve its predictive capabilities.

6. Finally, the model has been utilized in determining the degree of treatment required by the waste treatment plants discharging into the Potomac Estuary to meet the water quality standards adopted by the District of Columbia and the State of Maryland. These results have been presented to the conference by FWPCA as recommendations.

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